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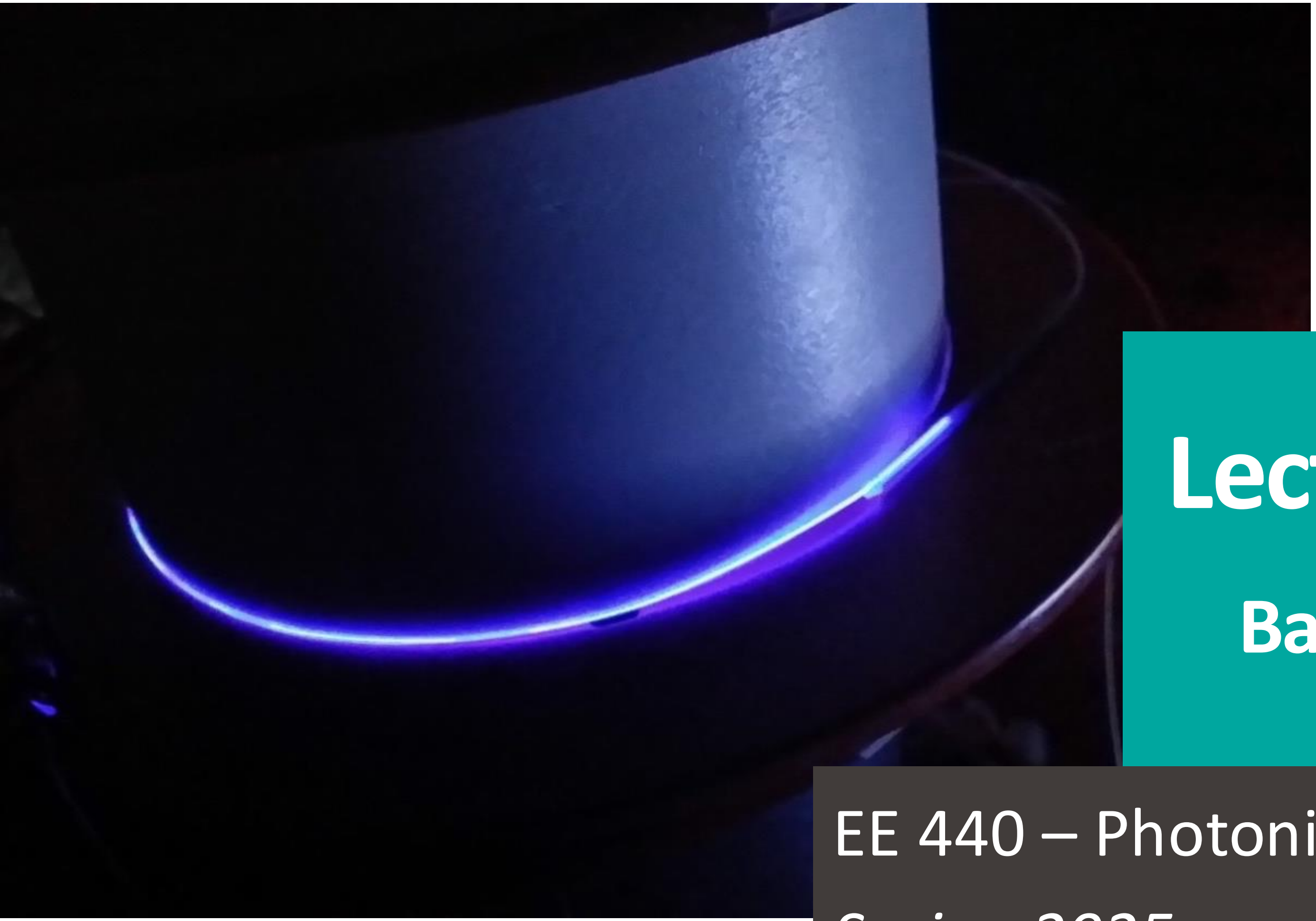

Lecture 11

Basics of optical amplification

EE 440 – Photonic systems and technology

Spring 2025

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Lecture 11 outline

Intro to light amplification

- General concepts
- Amplifiers/amplification types

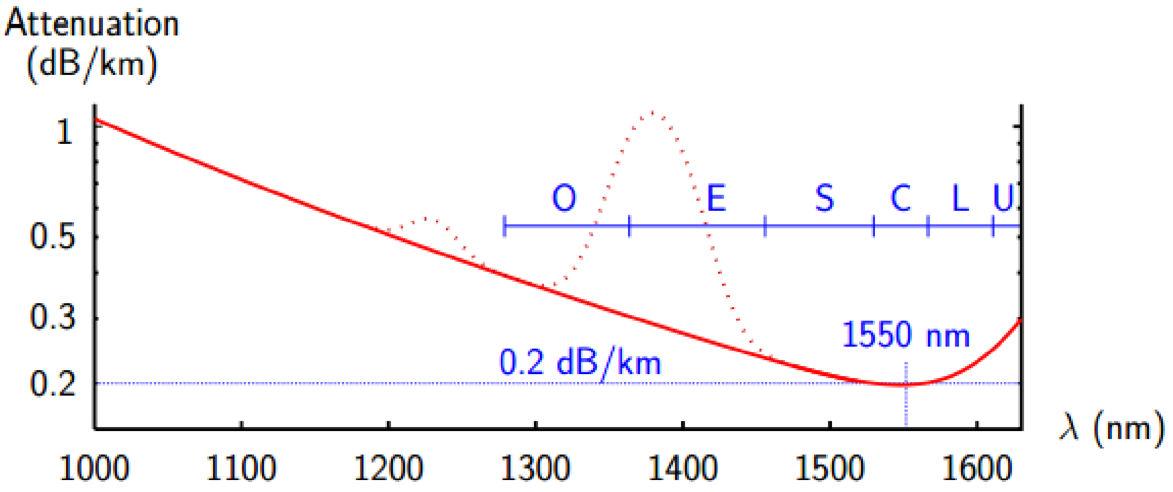
Amplifier properties

- Gain
- Saturation
- 3 levels and 4 levels pumping schemes

General concepts

Light amplification

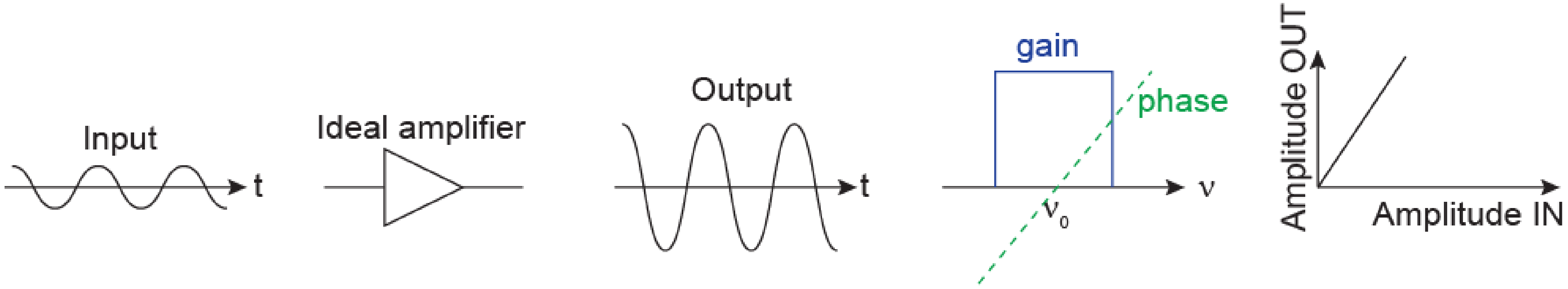
Limiting factor of high speed optical communication: attenuation



Can use optical amplifiers to avoid signal regenerators (O-E-O conversion)

Transmission bandwidth = amplifier’s gain bandwidth

Properties of ideal amplifier



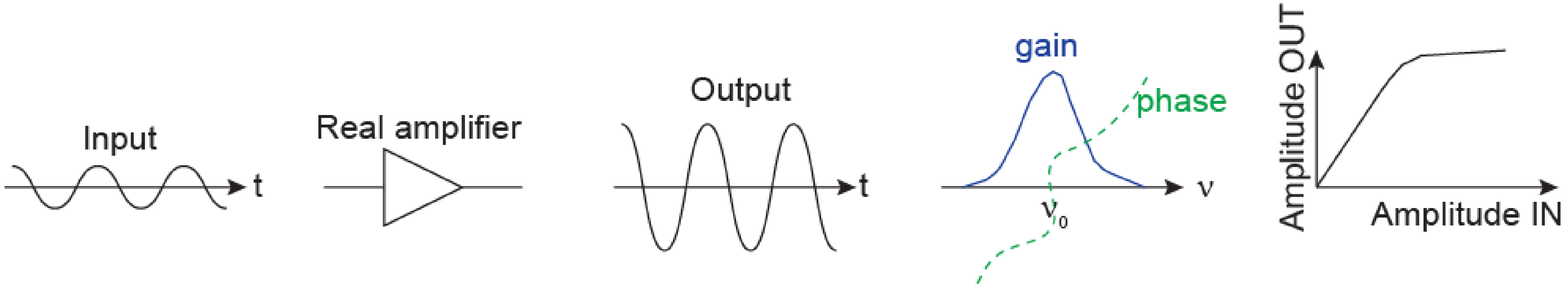
Linear system: amplitude of input signal increased by fixed factor

- Called the amplifier gain.

Gain is constant for all frequencies within the amplifier spectral bandwidth.

Amplifier may lead to phase shift, varies linearly with frequency (corresponds to time delay).

Properties of real amplifier



Gain and phase shift are frequency dependent .

- Constitute the amplifier transfer function.

May exhibit saturation: nonlinear behavior, output fails to increase in proportion to the input.

Introduce noise, randomly fluctuating components are always present at the output.

Benefits and requirements of optical amplifiers

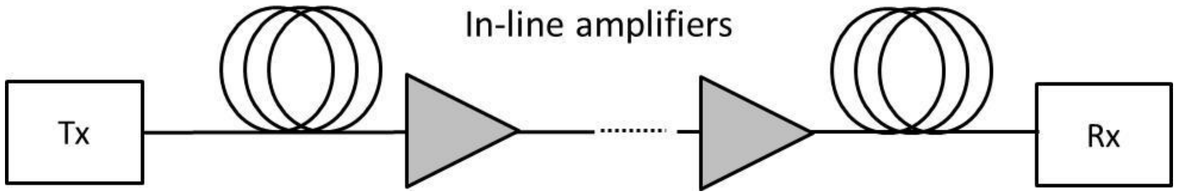
Benefits

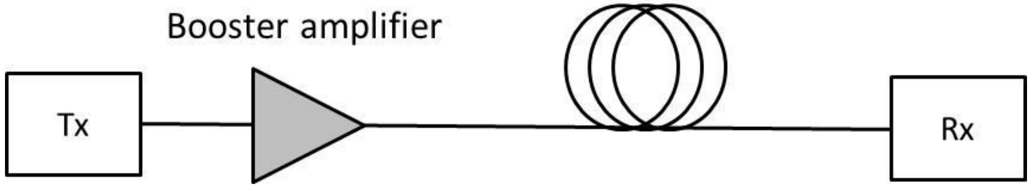
- Eliminates need for optoelectronic regenerators in loss limited systems
- Can improve the receiver sensitivity
- Can increase the transmitted power
- Transparent to bit rates and modulation formats
- Can amplify many WDM channels simultaneously

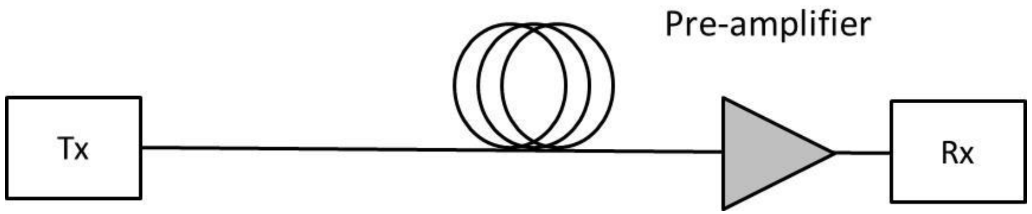
Requirements (ideal)

- High gain, high output power, high efficiency
- Large bandwidth
- Polarization insensitive
- Low noise
- No crosstalk between WDM channels
- Amplify broadband analog and digital signals, from kHz to 100’s of GHz

Amplifier applications







- In-line: compensates for transmission losses.
- Power-amp (booster) : increases the transmitter output power.
- Pre-amp: enhances the sensitivity of the receiver.

(some) Amplifier types

Semiconductor optical amplifiers (SOA) are electrically pumped

- Similar to semiconductor laser without feedback and biased below threshold.
- Lumped amplification

Doped fiber amplifiers use excitations of ions in the host fiber, they are optically pumped

- Erbium doped fiber amplifier (EDFA) is the most common.
- Lumped amplification

Raman & Brillouin amplifiers use nonlinear processes to transfer energy from a pump wave to a signal wave.

- Involve vibrations (phonon) of the silica glass.
- Distributed amplification

Parametric amplifiers use nonlinear process to transfer energy from the pump wave to a signal.

Lumped amplification

The optical power decreases as it propagates:

$$P(z) = P_{in} \exp(-\alpha z)$$

With amplifier spacing L_A, gain G of the amplifier is typically adjusted to compensate the link loss:

$$G = \exp(\alpha L_A)$$

- Typical amplifier spacing is 30 – 100 km
- The spacing must not necessarily be uniform

Distributed amplification

Let the local gain be $g_0(z)$, we get:

$$\frac{dp(z)}{dz} = [g_0(z) - \alpha]p(z)$$

Ideally we would want $g_0(z) = \alpha$

- But pump power is not constant: gain decreases with distance from pump source

Condition for compensation over distance L_A is:

$$\int_0^{L_A} g_0(z)dz = \alpha L_\mathrm{A}$$

Amplifier gain

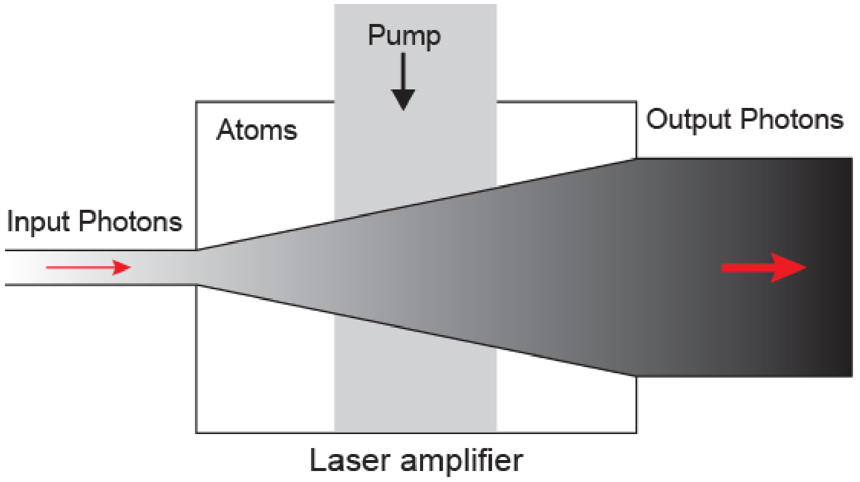
Basic concepts

Optical amplification is based on stimulated emission:

- Same mechanism used in lasers
- Amplifier = laser without feedback

Main requirement

- Optical gain realized when the amplifier is pumped (optically or electrically) to achieve population inversion



Optical gain depends on

- Frequency of the incident signal
- Local beam intensity at any point inside the amplifier

Recall of photon-atom interaction

Spontaneous emission

- Atom is in the upper level
- Emits photon independently of presence of other photons
- Responsible for NOISE in amplifiers

$$W_{sp} = \frac{1}{\tau_{sp}}$$

Stimulated emission

- Atom is in the upper level
- A photon may force the emission of a clone photon
- Responsible for GAIN in amplifiers

$$W_{stim} = \phi\sigma(\nu)$$

Absorption

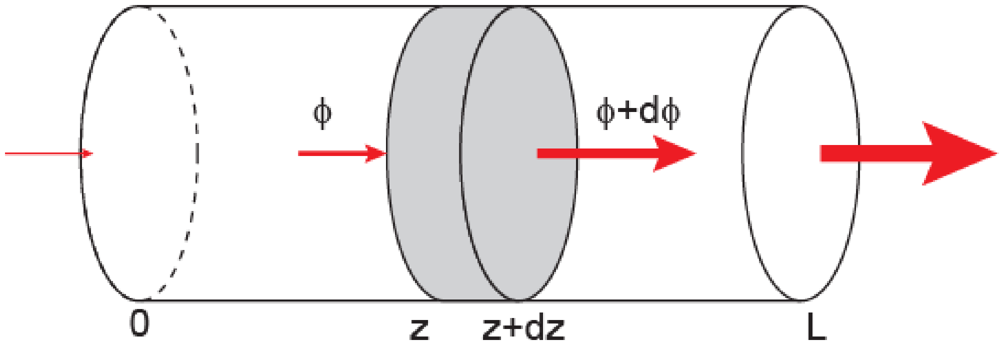
- Atom is in the lower energy level
- Photon may be absorbed
- Responsible for SATURATION in amplifiers

$$W_{abs} = \phi\sigma(\nu)$$

Amplifier gain

Assume medium with per unit volume N_1 atoms on lower level and N_2 atoms on upper level

- Let's consider the increase in photon flux $d\phi$ over distance dz



- Photons gained by stimulated emission: $\frac{dn_{ph}}{dtdV} = N_2 W_{stim} = N_2 \phi\sigma(\nu)$

- Increase in photon flux: $d\phi_{stim} = \frac{dn_{ph}}{dtdV} dz = N_2 \phi\sigma(\nu) dz$

- Decrease in photon flux: $d\phi_{abs} = N_1 W_{abs} dz = N_1 \phi\sigma(\nu) dz$

Amplifier gain con'd

$$d\phi = d\phi_{stim} - d\phi_{abs}$$

$$d\phi = (N_2 - N_1)\phi\sigma(\nu)dz$$

$$\frac{d\phi}{dz} = \gamma(\nu)\phi(z)$$

With $\gamma(\nu) = (N_2 - N_1)\sigma(\nu)$ *Gain coefficient*

$$\phi(z) = \phi(0)\exp[\gamma(\nu)z]$$

Photon flux increases exponentially with distance in a gain medium with gain coefficient $\gamma(\nu)$

- Value of $\gamma(\nu)$ depends on strength of the population inversion $(N_2 - N_1)$ and material properties linked to the emission cross section $\sigma(\nu)$

Amplifier gain con'd

The optical intensity is linked to the photon flux through:

$$I(z) = h\nu\phi(z)$$

Can therefore also write the evolution of the intensity as:

$$I(z) = I(0)\exp[\gamma(\nu)z]$$

For an interaction region of length L, overall amplifier gain $G(\nu)$ defined as

$$G(\nu) = \frac{\phi(L)}{\phi(0)} = \exp[\gamma(\nu)L]$$

Amplifier gain bandwidth

The transition cross section between 2 levels can be expressed as: $\sigma(\nu) = \frac{\lambda_0^2}{2\pi\tau_{sp}} g(\nu)$

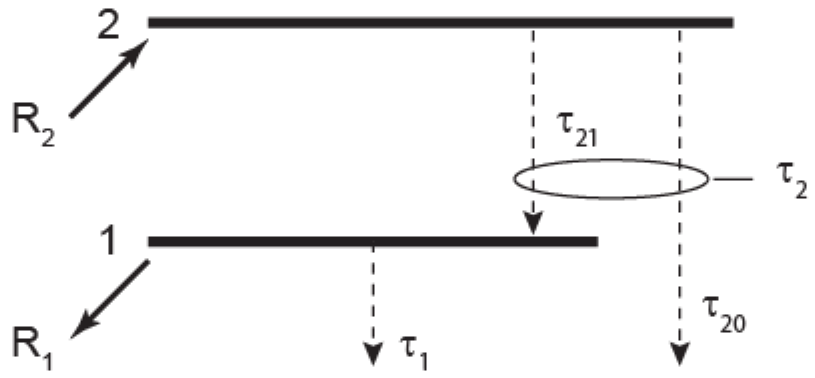
Recall that $g(\nu)$, the normalized lineshape function of the transition is:

- centered about atomic resonance $\nu_0 = (E_2 - E_1)/h$
- has a width $\Delta\nu$

Frequency dependence of amplifier gain G is due to proportionality to the lineshape function $g(\nu)$

Pumping of gain medium

Pumping a gain medium - amplifier budget in the absence of input signal



Level 1 and 2: overall lifetime τ_1 and τ_2, permitting transitions to all lower levels.

τ_2 has 2 contributions:

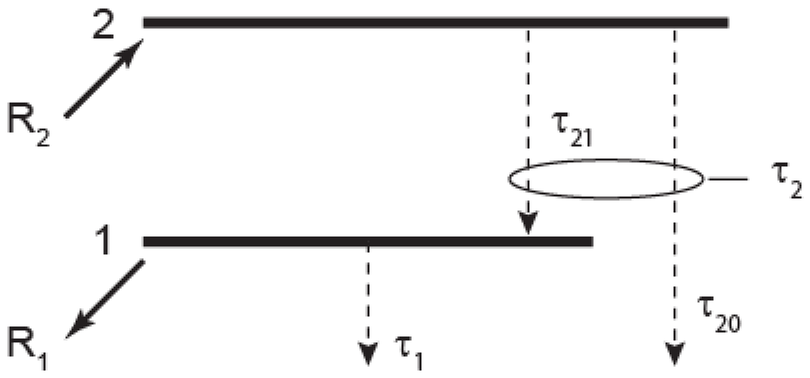
- One associated with decay from 2 to 1, τ_{21}
  - Contains both radiative τ_{sp} and non radiative τ_{nr} contributions.
  - In general $\tau_{sp} \ll \tau_{nr}$
- One with decay from 2 to all other lower levels, τ_{20}
  - In general $\tau_{21} \ll \tau_{20}$

$$\frac{1}{\tau_{21}} = \frac{1}{\tau_{sp}} + \frac{1}{\tau_{nr}} \Rightarrow \tau_{21} \approx \tau_{sp}$$

$$\frac{1}{\tau_2} = \frac{1}{\tau_{21}} + \frac{1}{\tau_{20}} \Rightarrow \tau_2 \approx \tau_{21}$$

$$\tau_2 \approx \tau_{sp}$$

Pumping a gain medium - amplifier budget in the absence of input signal



$$\frac{dN_2}{dt} = R_2 - \frac{N_2}{\tau_2}$$

$$\frac{dN_1}{dt} = -R_1 - \frac{N_1}{\tau_1} + \frac{N_2}{\tau_{21}}$$

At steady state the population difference (without input) $N_0 = N_2 - N_1$ is:

$$N_0 = R_2\tau_2\left(1 - \frac{\tau_1}{\tau_{21}}\right) + R_1\tau_1$$

Population difference without input

Typical time dependences

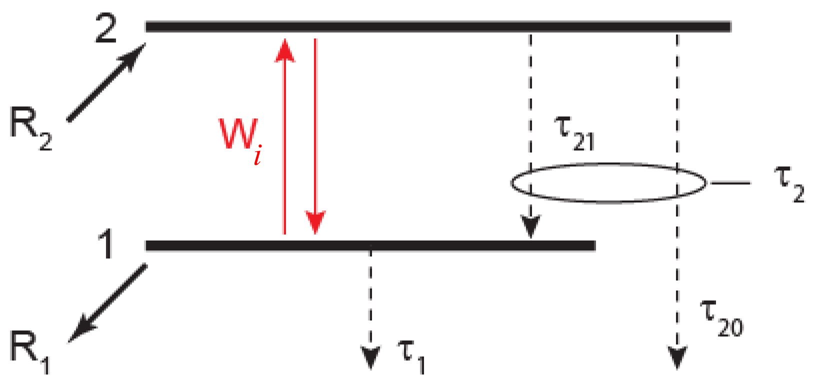
Since $\tau_2 \approx \tau_{sp}$

And since to reach large population difference need short lifetime on the lower level (as to limit population N_1), meaning we want $\tau_1 \ll \tau_{sp}$

$$N_0 \approx R_2\tau_{sp} + R_1\tau_1$$

Approximated population difference without input

Pumping a gain medium - amplifier budget with an input signal



$$\frac{dN_2}{dt} = R_2 - \frac{N_2}{\tau_2} - N_2 W_i + N_1 W_i$$

$$\frac{dN_1}{dt} = -R_1 - \frac{N_1}{\tau_1} + \frac{N_2}{\tau_{21}} + N_2 W_i - N_1 W_i$$

At steady state condition the population difference $N = N_2 - N_1$ is:

$$N = \frac{N_0}{1 + \tau_s W_i}$$

Population difference with input

$$\tau_s = \tau_2 + \tau_1 \left(1 - \frac{\tau_2}{\tau_{21}}\right)$$

Saturation time constant

Saturation photon flux density

Recall that $W_i = \phi\sigma(\nu)$

$$N = \frac{N_0}{1 + \tau_s W_i}$$

$$N = \frac{N_0}{1 + \tau_s \phi\sigma(\nu)}$$

$$N = \frac{N_0}{1 + \frac{\phi}{\phi_s(\nu)}}$$

with $\phi_s(\nu) = \frac{1}{\tau_s \sigma(\nu)}$

Saturation photon flux density

For homogeneously broadened media

$$\gamma(\nu) = N\sigma(\nu)$$

$$\gamma(\nu) = \frac{N_0}{1 + \frac{\phi}{\phi_s(\nu)}}\sigma(\nu)$$

$$\gamma(\nu) = \frac{\gamma_0(\nu)}{1 + \frac{\phi}{\phi_s(\nu)}}$$

with $\gamma_0(\nu) = N_0\sigma(\nu)$

Small signal gain coefficient

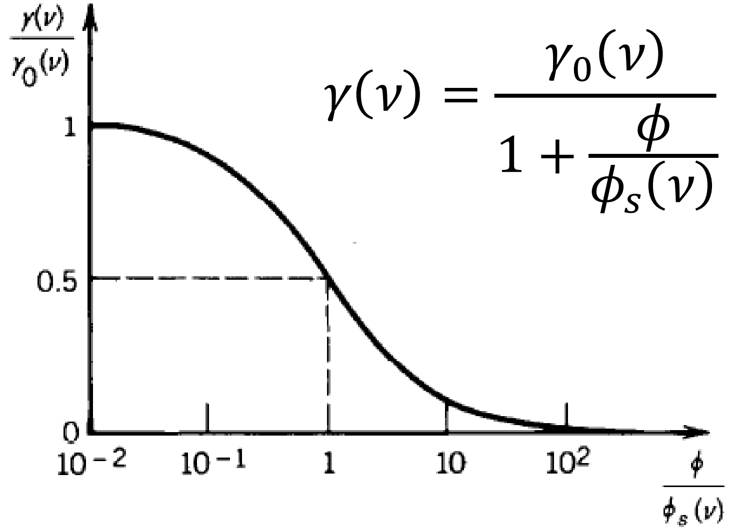
Saturated gain

For *weak radiation* (i.e. small signal approximation):

$$\gamma \approx \gamma_0 \text{ and } G = G_0 = \exp(\gamma_0 L)$$

Gain coefficient is a decreasing function of photon flux.

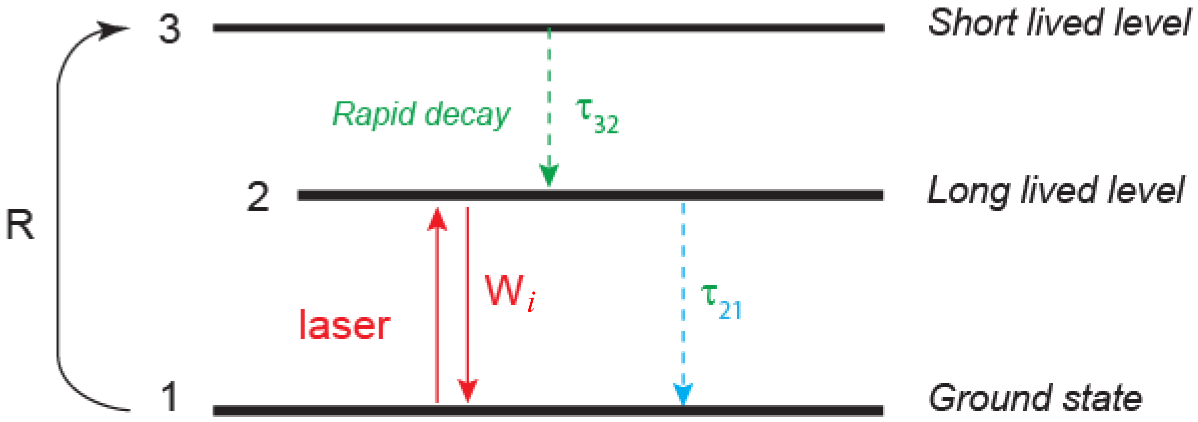
- At saturation photon flux density $\phi_s(\nu)$, gain is decreased by half.
- Saturation is due to the fact that at high flux, absorption and emission dominate the interaction.



$$\gamma(\nu) = \frac{\gamma_0(\nu)}{1 + \frac{\phi}{\phi_s(\nu)}}$$

Pumping schemes

3-level optical pumping schemes



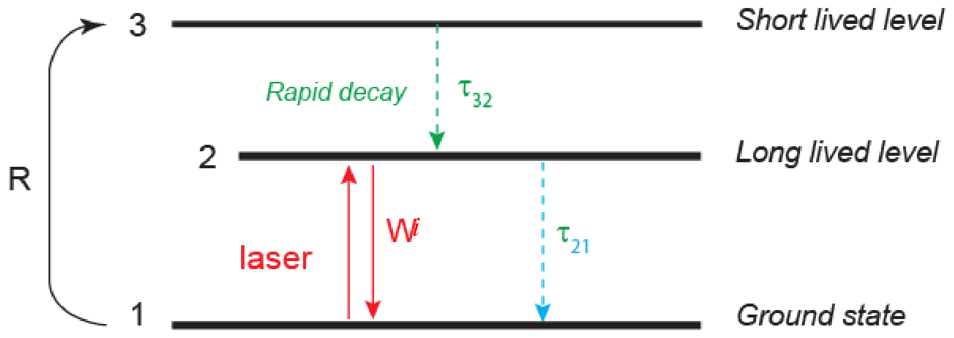
Amplification takes place between level ‘1’ and level ‘2’

- Ground state is the lowest amplifier level ‘1’

Population inversion is accomplished by making use of energy level(s) lying above level ‘2’

- Designated as level ‘3’
- Transition between levels ‘3’ and ‘2’ has short lifetime → decay occurs rapidly → population does not accumulate at level 3 ((i.e. $N_3 \approx 0$)
- 3-1 decay is slow ($\tau_{32} << \tau_{31}$)

3-level systems con'd



Under *rapid 3-2 decay* the 3-level system is a special case of previously presented analysis:

$$R_1 = R_2 = R \qquad \tau_1 = \infty \qquad \tau_2 = \tau_{21}$$

Let's return to the rate equation in the presence of signal:

$$\frac{dN_2}{dt} = R - \frac{N_2}{\tau_{21}} - N_2 W_i + N_1 W_i$$

Let N_a be the total population: $N_a = N_1 + N_2 + N_3 \approx N_1 + N_2$

3-level systems con'd

Want to express population inversion $N = N_2 - N_1$ in standard form of:

$$N = \frac{N_0}{1 + \tau_s W_i}$$

By solving the rate equation together with the expression for N_a we get:

$$N_0 = 2\tau_{21} R - N_a$$

Population difference without input

$$\tau_s = 2\tau_{21} \approx 2\tau_{sp}$$

Saturation time constant

3-level system conclusions

To attain population inversion ($N > 0$, i.e. $N_0 > 0$) requires:

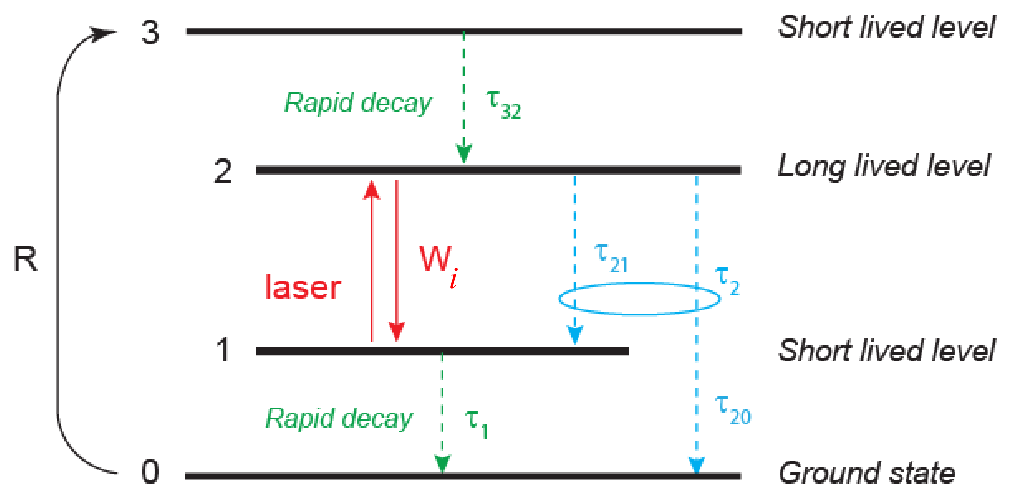
$$2\tau_{21}R - N_a > 0 \Rightarrow R > \frac{N_a}{2\tau_{sp}}$$

Just to make $N_2 = N_1$ (i.e. $N_0 = 0$) requires a substantial pump power density, given by:

$$(E_3 - E_1)\frac{N_a}{2\tau_{sp}}$$

- The large population in the ground state (which is the lowest laser level) provides an inherent obstacle to achieving a population inversion in a three-level system

4-level pumping schemes



Amplification occurs between level ‘1’ and level ‘2’

- Level ‘1’ lies above ground state: at equilibrium will be virtually unpopulated provided that $E_1 \gg k_b T$

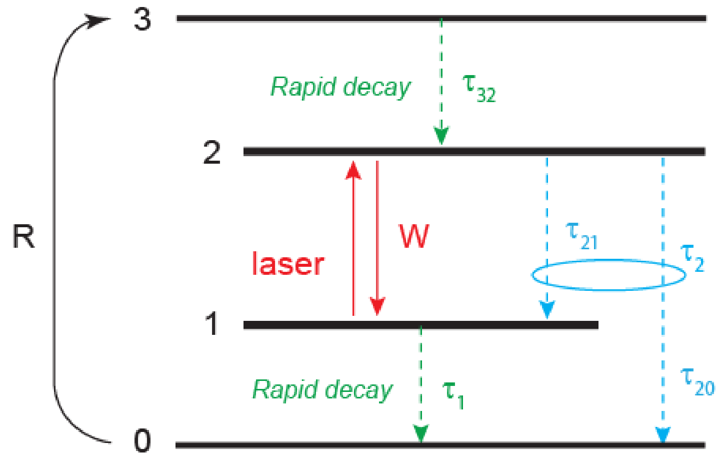
Pumping accomplished by making use of the energy level(s) above level 2 and designated as level 3

- Transition between levels ‘3’ and ‘2’ has short lifetime → decay occurs rapidly → population does not accumulate at level 3 (i.e. $N_3 \approx 0$)
- Level 2 is long lived and level 1 is short lived

4-level systems con'd

External source of energy pumps atoms from level 0 to 3 at a rate R

- Decay 3 – 2 considered instantaneous
- Pumping level 3 equivalent to pumping level 2: $R_2 = R$
- Atoms neither pumped in or out of level 1: $R_1 = 0$
- Non radiative decay is negligible: $\tau_{21} \approx \tau_{sp}$
- Typical lifetimes: $\tau_{20} >> \tau_{sp} >> \tau_1$ ➔ $\tau_2 \approx \tau_{sp}$ and $\tau_2 >> \tau_1$



$$N = \frac{N_0}{1 + \tau_s W_i}$$

$$N_0 = \tau_{sp} R$$

Population difference without input

$$\tau_s \approx \tau_{sp}$$

Saturation time constant

4-level systems conclusions

No minimum pumping requirement for an ideal four-level system as level 1 is initially empty:

- Since only a few atoms must be excited into upper laser level to form a population inversion, a four-level laser is much more efficient than a three-level one.

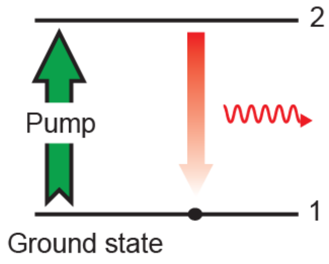
Saturation time of 4-level system is shorter than for 3-level system

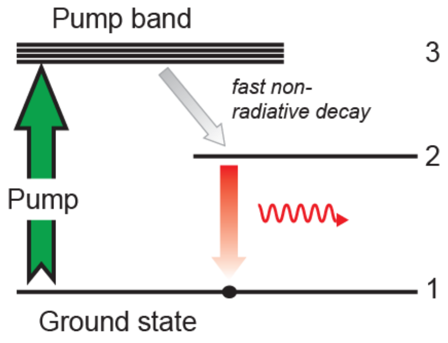
- Four-level system therefore has a higher saturation flux

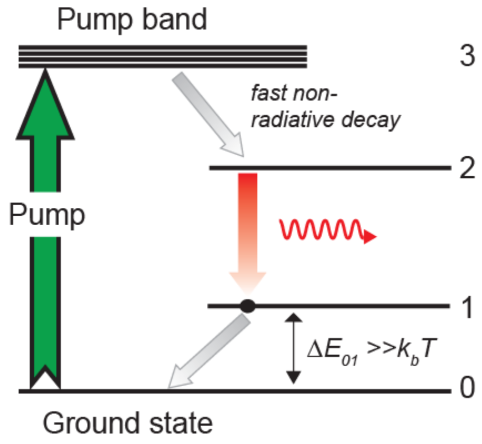
$$(\tau_s)_{4level} < (\tau_s)_{3level}$$

$$(\phi_s)_{4level} > (\phi_s)_{3level}$$

Summary of pumping schemes







2-level system

- (optical) pump depletes population of upper state ✗
- Cannot achieve population inversion ✗

3-level system

- Efficient pumping into pump band ✅
- Fast decay into upper laser level ✅
- Pump half of the total population from lower to upper for inversion ✗
- High pumping threshold ✗

4-level system

- Efficient pumping into pump band ✅
- Fast decay into upper laser level ✅
- Fast depopulation of the lower level ✅
- Population inversion easily achieved ✅
- Low pumping threshold ✅